E. Andrew Boyd presents us with a wonderful ride through some of the great riddles of the ages. He does so to elaborate his "Beyond Comprehension" conjecture—that realities exist beyond our hope of ever comprehending them. That conjecture is at once obvious, while it screams out for our denial. And the very fact that it cannot be proved either way only adds to its validity. All the while we have his examples—Oh the fine mind-teasing examples!

Dr. John Lienhard
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When I was young, I was drawn to mathematics by concepts that were beyond ordinary comprehension. Boyd's book explains limits to human comprehension in a way that is accessible to a broad audience. In the spirit of *Freakonomics*, and books by Malcolm Gladwell, Bill Bryson, and Neil deGrasse Tyson, Boyd inspires a sense of wonder about the outer limits of comprehension in mathematics and physics.

Dr. L. Ridgway Scott
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Beyond Comprehension

A Scientific Look at the Challenge of Knowing Everything

E. Andrew Boyd

Hamilton-Haverbrook
Worldwide

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A Scientific Look at the Challenge of Knowing Everything

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Preface

A teacher stands before a classroom of students who are busily taking notes.

"And in my view, Jefferson's defense of these basic rights lacked conviction. Any discussion?"

Silence. The teacher continues.

"Let me just add that personally I believe the Bill of Rights to be a silly, inconsequential recapitulation of truths already found in the Constitution. Any comment?"

More silence.

"No, *scratch* that! The Constitution *itself* should never have been ratified! It's a dangerous document! All power should rest with the *executive!* What do you think of *that!*"

Even more silence.

"JEFFERSON WAS THE *ANTICHRIST!* DEMOCRACY IS FASCISM! BLACK IS WHITE! NIGHT IS DAY!"

"Boy, this course is getting interesting," says one student to another.

"You said it," comes the response. "I didn't know half this stuff."

This *Doonesbury* comic strip first appeared in 1985 and spoke to sixties-educated teachers who were confronting a new generation of students; a generation that seemed more interested in grades than in changing the world. How could students be so blind and complacent that equating black and white failed to evoke incredulity?

As humankind charges headlong into the future, I sometimes feel as though we, too, are spending so much time taking notes that we're failing to appreciate the world we're confronted with. Throughout most of human history the universe was filled with mystery. Where did the sun go at night? Why did humankind hold a special place in the universe? With advances in physics, biology and the other sciences, the world's certainly become a much less mysterious place. Yet, as we probe ever more deeply, we find we're still facing puzzles that defy explanation. We laugh, make note of them, and file them away in the recesses of our brains. But we shouldn't. We should marvel at them. In *Beyond Comprehension* I dust off some of life's special puzzles in an effort to evoke a combination of incredulity and astonishment.

None of the topics we'll look at are altogether new. In fact, many have entire books written about them, and I encourage readers who encounter something new and exciting to follow up with one of the references I've provided. My intent isn't to delve too deeply into any one topic but to share the wonder found in many.

To achieve this sense of wonder requires help. All of the topics have been chosen for their enigmatic nature. But while some tend to jump off the page, others require reflection. In some cases the biggest challenge may be familiarity. For example, I've observed that many people have trouble getting excited about gravity. It's such a part of our daily lives that it takes effort to break free of our mind-set and appreciate how remarkable gravity really is and why Newton's theory of gravity was once viewed with great skepticism.

Another challenge is the natural human desire to explain things. Explanations are important in that they help as we try to make sense of something new that we encounter. But explanations aren't a requirement, especially when they're not forthcoming or miss the fundamental point. It's okay to simply stop and smell the roses.

A few housekeeping items. All chapters stand independently except for "Reflections," which is why I've included all notes, references, and appendices with each chapter rather than at the end of the book.

Also, I've done my best to use as little math as possible, but I couldn't bring myself to shy away from it altogether. For those who are interested, the mathematical details provide another level at which to marvel over the perplexities of our world. If in the course of reading you encounter something technical that doesn't interest you, skip it. Doing so shouldn't hamper your progress. For the record, I've used nothing beyond high school math in the pages that follow, and most of the time nothing beyond elementary school math. Reading the first and last chapters is also an option, returning to the intermediate chapters when time and interest allow.

I wish you a happily befuddling journey on your road to discovery.

E. Andrew Boyd

It is of great use to the sailor to know the length of his line, though he cannot with it fathom all the depths of the ocean.

– John Locke

Chapter One

No Mystery: The Limits of Human Comprehension

Dogs are remarkable creatures. They display emotion when playing or cuddling with their owners. They form complex relationships with people and other animals. And they're keenly intelligent, capable of learning hundreds of words, from "sit" to "fetch." But there's something pretty basic dogs don't grasp.

The number ten.

And they're not alone. Ten is a foreign concept to all animals other than humans. Research has shown that many animals are able to discern when one set of things is larger than another, and some animals demonstrate a distinct knowledge of the numbers one, two, and three. But go much beyond that and the animal mind withers to the task. It's not that the dog mind doesn't understand the concept of ten; that it somehow recognizes the digits on two human hands as "something" but doesn't know what to do with that "something." The specific concept simply doesn't materialize. The dog brain isn't wired to wrap itself around "ten." Dogs and other animals don't fail to understand ten. Ten is *beyond their comprehension*.

Yet for humans the number ten is so elementary we take it for granted. Ten comes after nine and before eleven. Two fives make ten. Ten plus two equals twelve. All of these simple observations—much less the edifice of mathematics that underlies so much of our modern technology—are completely beyond the mental capacity of animals.

Mathematics is just one realm where the human mind comprehends what the animal mind cannot. You and I might agree to meet downtown at a coffee shop next Tuesday at 7:00 a.m., and barring forgetfulness or an emergency, we'll drive our cars and get together at the appointed time. The animal mind would be confounded on many fronts, from understanding the rules of the road to the task of establishing a specific meeting time. Again, the issue is not a lack of understanding but a lack of comprehension. It seems some animals have a vague sense of time in the same way they have a vague sense of bigger and smaller numbers, but even that's open to debate. Many researchers believe animals truly "live in the moment"—that they have no sense of past or future.

Yet time comes so naturally to us that we rarely give it a second thought. And we certainly don't live our lives in the moment. We remember lessons from the past and plan for the future—often causing ourselves unnecessary stress in the process.

The point is not to dwell on the cognitive prowess of humans, though the human mind, stemming from the human brain, is extraordinary when compared to that of other animals. The point is simply to provide background for the following widely accepted observation:

> ***Observation:*** *There are real things in our world that dogs and other animals can't comprehend.*

It's worth emphasizing two very important points. First, it can't be overstated that *can't comprehend* is different from a mere lack of understanding. I may not understand how an automobile engine works, but if I took enough time, I could learn. And even without knowing the details, I know that when gasoline burns, it releases energy, which the engine harnesses in a way that makes a car move. I can comprehend what an engine is even if I don't fully understand it. To say that animals can't comprehend numbers means something much stronger. It means they don't even know what they're missing.

The second point of emphasis is the use of the term *real things*. We're not talking about something fanciful. The two examples just mentioned are the existence of numbers and the passage of time. Even if all human life were extinguished, the number ten would still be ten and time would keep marching forward. This is the sense in which we use the word "real."

Our observation sets the stage for the following conjecture:

> ***Beyond Comprehension (BC) Conjecture:*** *There are real things in our world that humans can't comprehend.*

The foundation for this conjecture is quite simple: humans are animals, too. Why should we, as a species, expect that we're wired to comprehend everything? No other animals can. Has evolution singled us out, taking us to a pinnacle from which we're able to comprehend all that is? If anything, evolution suggests just the opposite. Every animal we look at, at least for the time being, has reached a cognitive plateau. We know they haven't reached a pinnacle since we can look at our own cognitive abilities and see that other animals still have a long way to go. Extrapolating, it seems far, far more likely that we, too, have reached a plateau and not a pinnacle.

Fully absorbing the conjecture takes some reflection. Our perspective on the world is through that which we can comprehend. How can there be real things we can't comprehend? Or even if there are, why should we care? After all, they're beyond our comprehension, which means we'll never be able to wrap our heads around them in any meaningful sense. Whether they're real or not, since we can't comprehend them, they're irrelevant—at least from the perspective of rational, scientifically minded humans.

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Comedian George Carlin was born an Irish Catholic, though he was quick to point out that by the time he'd grown up, he was an American. He attended his parish school in New York, a school that was traditional in its teaching of church doctrine but progressive in its educational methods: uniforms weren't required, no grades were given, and classes weren't segregated by sex. Unlike the dictatorial, ruler-toting environment Catholic schools were known for, the school created an environment of freedom. So much freedom, according to Carlin, that by eighth grade many of the students had lost the faith. Students were taught to raise questions—questions that sometimes evoked a response if not an answer: "It's a mystery."

Mystery is a loaded word. It can be used in many subtly different ways, but when it comes to expressing our understanding of something or lack thereof, it carries with it a sense of throwing in the towel. Take the statement "life's a mystery." The intent is to convey a sense that life is more than an organized collection of molecules capable of reproduction. Given our personal experience with our own lives, it's a justifiable belief. But the use of the word "mystery" evokes a sense that there's nothing more that can be said. We can seek to "experience" or "enter into" the mystery, but we can't hope to come to terms with it in any rational sense. Consider the following short-lived discussion between a scientist and a Christian theologian.

Scientist: How do you explain the doctrine of the Trinity, that God, Jesus, and the Holy Spirit are both three distinct beings and one being at the same time?

Theologian: It's a mystery.

In claiming the doctrine to be a mystery, the theologian has brought rational discussion to a halt.

By contrast, scientists avoid the word mystery because it implies something that can't be reckoned with through reason. The wave/particle duality of matter—where subatomic particles sometimes act like waves and at other times like particles—is as bewildering to physicists as the Trinity is to theologians. It's also perplexing, astounding, baffling, puzzling, head-scratching, confounding, and astonishing. But it's not a mystery, because the word mystery carries unwanted baggage: an implied "that's all, folks."

Mystery isn't a bad thing. Rocking a newborn. Watching the stars rise against a backdrop of remote mountains. Unexpected euphoria during a quiet moment. Each inspires a sense of mystery. Words only detract from the visceral resonance of the moment. But as we embark upon our quest to discover shadows of the incomprehensible, it's important to avoid any confusion. We are not in search of mysteries—of anything that brings an end to rational discussion. Our goal is to see if our very rationality leads us to inexplicable realities—paradoxes—or, at the very least, realities so counter to intuition they leave us dumbfounded. We're then left to ask if these inexplicable realities are telling us something. Are they shadows of things beyond comprehension?

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Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity.

—Sir Isaac Newton

Chapter Three

Gravity

Here's a magic trick you can easily perform for friends. Take a favorite stuffed animal and place it at one end of your kitchen table. Next, take a seat across the table and out of reach of the animal. Now proclaim to everyone present that you will cause the animal to slide across the table and into your hands. Stare intently at it. Stare some more. Build the suspense. Finally, after a good ten seconds, look your friends in the eye and proclaim, "What? Are you crazy? I can't magically make it move without touching it." To make your point, walk over to the animal and carry it back to your seat.

Now you're ready for the payoff. With everyone nodding in agreement, raise the animal in front of you and let it go. Without any visible means of propulsion, the animal drops from your hands to the floor. Gravity, it seems, is a magical force.

When first presented with this trick, most people are unimpressed. We're so familiar with gravity that it doesn't seem especially magical to see something drop to the floor. We'd be surprised if we let something go and it didn't drop. Yet we shouldn't allow our familiarity with gravity impede our appreciation for something scholars struggled with for centuries.

One of the great turning points in all of history arrived with Isaac Newton's theory of gravity, published in his masterwork the *Principia* in 1687. Decades earlier, Johannes Kepler carried out laborious calculations that led him to discover remarkable geometric facts about the movement of the planets. Remarkable as they were, however, they were simply facts, devoid of any underlying scientific theory to explain them. Kepler's discoveries proved fodder for many scholarly inquiries, but it was Newton who ultimately put the pieces together in a mathematically rigorous way. Newton not only developed a theory of gravity but went on to develop a general theory of motion that we now refer to as classical, or Newtonian, mechanics. Gravity became a special case of Newton's theory of motion, but one with a special twist.

At the foundation of classical mechanics are Newton's three laws of motion, the first of which is the following:

> *Newton's First Law of Motion:* An object at rest will remain at rest unless acted upon by an external force. An object traveling in a straight line at a constant speed will continue to do so unless acted upon by an external force.[1]

Examples are easy to cite. A kitchen chair will remain motionless until someone or something moves it. A billiard ball will continue across a billiard table in a straight line and at constant velocity until it encounters a cushion or another ball. Notice that here, as in most examples that jump to mind, something physical actually causes the

[1] The second assertion, that an object traveling in a straight line at a constant speed will continue to do so unless acted upon by an external force, represented a significant step forward in our understanding of motion. Day-to-day experience tells us the natural state of an object is stationary. If I give a box a shove along the floor and then let go, it quickly comes to rest. Newton's first law tells us the opposite: the box should keep moving unless acted on by an external force. We now recognize this force as friction, but for most of human history, it was assumed that force was required to keep an object moving because the natural state of an object was at rest. Even today children grow up making this assumption since it works in practice.

movement. I may pick up and move the chair, or it may be knocked over by a carelessly wielded vacuum cleaner, or it may be pushed across the floor by a strong gust of wind. It's natural to equate force with something physical—me, a vacuum cleaner, or molecules of air. And it seems mystical to imagine applying force to an object without something actually touching it.

Yet when it came to gravity, Newton appeared to be postulating just such a mystical entity. Here was the twist: the external force that kept the planets orbiting the sun rather than traveling off in a straight line was…what?

The idea of such a mystical force was met with incredulity by many. Christiaan Huygens dismissed the underpinnings of Newton's work as "absurd."[2] Gottfried Wilhelm Leibniz called Newton's gravity a "return to occult quantities, and even worse, to inexplicable ones."[3] Scholars—natural philosophers—were seeking to understand nature on its own terms, not through deference to something as unnatural as Newton was proposing.

Complicating the situation was the rise of what came to be known as mechanical philosophy. For centuries natural philosophy was dominated by the theory of substantial forms. The theory traces its roots to Aristotle, though it was refined and embraced by Scholastics beginning in the twelfth century. The theory was convoluted, expansive, and differed greatly in detail from one articulation to another. But a core precept was that an entity's substantial form was that which made it what it was—its essence. Within this context it would be perfectly acceptable to claim that a physical object, when dropped, fell to earth simply because that was its nature.

By the time of Newton, the theory of substantial forms was not only discredited in many circles but viewed with disdain. Descartes

[2] Brian Clegg, *Gravity: How the Weakest Force in the Universe Shaped Our Lives* (New York: St. Martin's Press, 2012), 73.
[3] Ibid.

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What can be said at all can be said clearly, and what we cannot talk about we must pass over in silence.

—Ludwig Wittgenstein

Chapter Eight
Reflections

The universe as we know it runs according to well-defined laws. If I throw a rock into the air with a given speed and direction so that it lands thirty feet in front of me, then do so again, the rock will again land thirty feet in front of me. That's the very notion of a natural law. But why, we might ask, thirty feet? Why doesn't the force I apply cause the rock to land twenty feet away? Or forty feet?

The rock lands where it does because of certain constants of nature—numbers underlying the equations that describe how gravity works. If we could change any of these constants while leaving the underlying laws alone, we would, in fact, see that the rock travels more or less than thirty feet following a path of similar shape. Why are the constants the values they are? We don't know. An explanation based on a deeper understanding of our physical universe would be very exciting, but it's not clear that such an explanation exists. For now, and possibly forever, we must content ourselves with simply measuring these important constants.

We can, however, ask the following question. If we were to change the values of any constants, would it significantly change how the universe works? An analogy is that of turning up a car radio. As we begin to increase the volume the music just gets louder, but at a certain point we blow out the speakers.

Physicists would typically expect that, like the volume on the radio, we can change these constants over a reasonable range without the universe acting radically differently. This in turn might lead us to ask the following intriguing question: how much can we change the constants of nature and still have a universe capable of supporting human life? If we dial the constants up or down, would human life still come about? Or if we so much as breathe on the dial, is it lights out?

It turns out that if we change some of the constants by only a hair's breadth, the universe becomes inhospitable to human life. In fact, a hair's breadth is a vast exaggeration. In the case of the cosmological constant, which influences the rate at which the universe is expanding, we can't move the dial by more than $1/10^{120}$. That's a frighteningly small number given that 10^{120} is trillions of trillions of trillions times bigger than the number of atoms in the known universe. And it raises a perplexing question: how is it that the big bang gave rise to such an exceedingly unlikely universe—a universe balanced in just the right way to support human life?

It's a question that's left physicists, philosophers, and theologians scratching their heads. Yet this seemingly impossible state of affairs can be resolved with a surprisingly simple argument. Consider the following syllogism.

> If the constants of nature were such that the universe couldn't support human life, then human life wouldn't exist.
>
> Human life exists.
>
> Therefore, the constants of nature are such that the universe can support human life.

The argument, a version of what's known as the *anthropic principle*, stands on its head the question about the precise balance of the constants of nature. Because we're here to ask the question, it shouldn't be surprising that the constants of nature support human life. If the universe didn't support human life, we wouldn't be here to ask it. We ask, therefore it is.[1]

The beauty of the anthropic principle isn't the depth of reasoning that lies behind it. Quite the contrary. It's been criticized as a tautology—a simple statement of fact leading nowhere. Okay, say its critics. We accept your argument. But it doesn't tell us anything about why the constants of nature are what they are. Isn't it still amazing that against all reasonable odds, we live in a finely tuned universe, a universe tuned in such a way as to support human life? To which proponents of the anthropic principle simply smile and shrug their shoulders.

The Beyond Comprehension (BC) conjecture—that there are real things in our world that humans can't comprehend—shares many similarities with the anthropic principle. Like the anthropic principle, most people have never thought about it. When they do, it seems both a bit eye-opening and self-evident at the same time, though it's not tautological.[2]

Another similarity is the propensity to dismiss the BC conjecture as interesting but nonetheless irrelevant. Things that are beyond comprehension are *beyond comprehension*. They're outside the realm of our minds' capabilities. Asking humans to do anything relevant in domains we can't grasp is like asking a dog to make sense of arithmetic. It may be fun to ponder, but we live in the world we perceive.

[1]Colleague Sarah Fishman made a connection between the anthropic principle and a game many children play when they ask the question, "What would I be like if my parents had never met?" Of course, it's a meaningless question, since if your parents hadn't met, the "you" to whom you're referring wouldn't be around to ask the question.

[2]There's nothing logically contradictory in arguing that the human mind is capable of understanding everything real, and the word "real" carries its own set of baggage.

Yet it's here that the similarity with the anthropic principle shines through: neither argument goes quietly. They're nagging and even a bit annoying, like the spouse who says I told you so. He or she may be right, but does it help to be reminded of the fact? In the case of the limitations of the human mind, most scholars would be hard-pressed to argue that the three pound chunks of organic matter we call our brains are up to the challenge of knowing everything. The BC conjecture isn't only tenable, it seems scientifically all but incontrovertible. But accepting the BC conjecture in practice is another matter. Doing so requires assessing whether an argument, however logically constructed, is built on a foundation that doesn't take into account some unknown realities. Who's to say when an argument is crossing into territory where it has no right to set foot?

Consider, for example, two commonly proposed resolutions of how the constants of nature came to be so finely tuned as to support human life. One is argument by design. An intelligent designer put the universe together in such a way that human life would spring up. Another is argument by chance. There exists a multiverse of many universes similar to ours growing out of their own big bangs. As time passes, each develops with its own different constants of nature. If we assume enough such universes, then one or more will evolve in such a way as to support human life. We just happen to live in one that does.

So what of these proposed resolutions? The first requires the existence of a designer of immense power—enough power to bring into existence not just the universe but a universe tuned to admit human life. The second relies on the existence of a vast, quite probably infinite collection of universes, only one of which we can observe.[3] Is either correct? Or is it possible we're simply in over our heads?

Another example surrounds claims regarding conscious life. Setting aside the problematic question of determining if a machine *is* conscious, we can ask if it's possible for a machine to *become* con-

[3] There are, of course, many variations on these themes.

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